Topology in and via Logic

Kick-Off Meeting

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Introduction

What is topology?

- An abstract mathematical study of shape and space.

Why topology?

- Applications in various areas of logic: algebraic logic, model theory, set theory, domain theory, formal epistemology, etc.
- Connection to MoL-courses, such as Mathematical Structures in Logic, Model Theory, and Topology, Logic and Learning.

Organization

Everything is (will be) available on the project website:

- https://tt0811.github.io/Tenyo/Topology2026.html

The first and second week:

- 6 lecture recordings from 2024 with slides provided
- 2 tutorials: on Friday (Jan 9 and 16) from 11:00 to 13:00 in F3.20
- 2 homework assignments: published on Jan 5 and 12, due on Jan 15 and 22
  - Completed in teams of up to 2 people (only one needs to submit via email)

The third week:

- 2 Q&A sessions: time TBA (Jan 19 and 23?)

The fourth week:

- Group presentations: given by teams (typically 2-3 persons per team)
  - With a guest lecture by Nick

A very naive introduction to topology

- Topology is a mathematical study of spaces.
- A space is a collection of points with shape.
- Mathematically, a topological space is a pair (X, τ), where X is a set and τ is a topology on X; X defines the collection of points and τ defines the shape.
- A topology on X is a collection of subsets of X.
- Let us take a look at the geometric intuition behind this definition.

  (See lecture notes and recordings for a mathematically proper introduction.)

A very naive introduction to topology

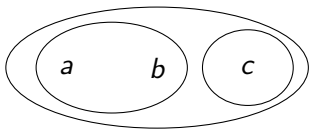
- Given a set X, what characterizes a shape of X?
- A naive answer: distance between points. (cf. metrix spaces.)
- More fundamentally, it is the farness and nearness between points.

A very naive introduction to topology

- Given a point x in the plane $\mathbb{R}^2$, what is the closest point to x?
- It is x itself.
- Why is x the closest point to x? (Try not to use distance.)
- Because, for any point $y \neq x$, there is a subset $U \subseteq \mathbb{R}^2$ such that $x \in U$ and $y \notin U$. It is not possible to separate x from x itself in this way.

A very naive introduction to topology

- We may say that the way X can be divided determines the farness and nearness between points, and thus the shape.
- This is represented by selecting a collection of subsets of X, namely, a topology.
- Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, \{c\}, \{a, b, c\}\}$.



- a and b are close because there is no set in τ separating them.
- a and c are distant because they are separable by sets in τ.

A very naive introduction to topology

- Finally, let us see two extreme examples. Let X be a set.
- Consider $\tau = \{\emptyset, X\}$.
  - In this topology, all points are close to each other.
  - That is why such τ is called the indiscrete topology or the trivial topology.
- Consider $\tau = \mathcal{P}(X)$, the power set of X.
  - In this topology, all points are far from each other.
  - That is why such τ is called the discrete topology.

Thank you!
See you all on Friday!