Topology and Falsification

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Outline

Background of Falsificationist Idea

Problems for Theory of science

- Finite observations - laws extend infinitely;
- Induction & truth;
- Demarcation of science and other knowledge

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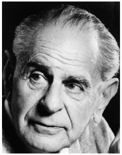
Example

All swans are white ($\forall s W s$).

Falsification as a solution



The criterion of falsifiability

A scientific hypothesis is falsified if an observation contradicts a consequence that can be deductively derived from it.[1]

Response to...

- Problem of induction
- Problem of demarcation

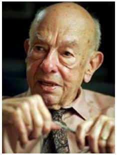
[1] C.f. Karl Popper, *The Logic of Scientific Discovery*, 1935

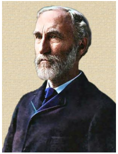
Newtonian Mechanics (NM)

- Failed to predict Uranus' orbit.
- Did not lead to a rejection of (NM)
- Instead: Doubt regarding number of planets in the solar system
- Subsequent discovery of Neptune
- Same strategy[a] failed for other celestial phenomena incongruent with (NM)
- Explanation by general relativity

[a] Only this time a planet called *Vulcan* was postulated.

Falsification

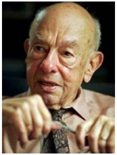
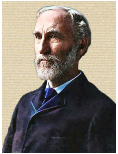




Weak Falsification

An (empirical) experiment can only test a hypothesis in conjunction with other assumptions.

Falsification

Weak Falsification

An (empirical) experiment can only test a hypothesis in conjunction with other assumptions.

Strong Falsification

An (empirical) experiment can test a hypothesis in isolation.

Epistemology × topology: why?

Points of contact

- Modelling common knowledge;
- More favourable axioms[a]
- Induction and infinite cases;
- Defeasible knowledge

[a] C.f. Aybüke Özgün, *Topological models for belief and belief revision*, 2013 (Master's Thesis).

Roadmap

Aims

- Formally elaborate on epistemic intuitions with specific objects encountered in the course
- Relate elaboration with discussion on weak vs. strong falsification
- Give an outlook on further research

Elaborating on Epistemic Intuitions

What Kind of Epistemological Framework?

Empiricism! (relativized to some subject area $\mathcal{S}$)

The only source of knowledge in $\mathcal{S}$ or for the concepts we use in $\mathcal{S}$ is experience.

Basic building blocks

- Observations $\mapsto$ Verifiable Propositions $\mapsto$ open sets
- Laws of Nature $\mapsto$ Falsifiable Propositions $\mapsto$ closed sets

Consider the latter as *extrapolations* of the former.

A Suitable Topological Structure?

Recall the set ω^ω and the topology τ generated by the basis $\mathcal{B}_\tau := \{C(s) \ : \ s \in \omega^{<\omega}\}$ with $C(s) := \{x \in \omega^\omega \ : \ s \lhd x\}$.

Proposal by Schulte & Juhl 1996

Rough proposal by Schulte & Juhl 1996

We have a sequence of observations $e, d, ...;$

E.g. e: observations 0 1 1 0

Hypothesis H, evaluated on data streams;

Form a topology on the collection of data streams

Verifiable and refutable with certainty

Binary Codes and Empirical Evidence

A Suitable Topological Structure?

Recall ω^ω and the topology τ generated by the basis $\mathcal{B}_\tau := \{C(s) \ : \ s \in 2^{<\omega}\}$ with $C(s) := \{x \in 2^\omega \ : \ s \lhd x\}$.

Empirical Encodings (cf. Schulte & Juhl, 1996)

Let 1 denote some concrete observation and set $0 := \neg 1$ to be any observation that does not conform with 1.

Example

Set $1 =$ "this swan is white" and $0 =$ "this swan is not white". Then the sequence 1^n encodes that n white swans have been observed.

Binary Codes and Empirical Evidence

A Suitable Topological Structure?

Recall the set ω^ω and the topology τ generated by the basis $\mathcal{B}_\tau := \{C(s) \ : \ s \in 2^{<\omega}\}$ with $C(s) := \{x \in 2^\omega \ : \ s \lhd x\}$.

Empirical Encodings

Let 1 denote some concrete observation and set $0 := \neg 1$ to be any observation that does not conform with 1.

Initial Segments and Extrapolations

Turning a (finite) sequence $d \in \omega^{<\omega}$ of empirical data into a law of nature amounts to infinitely extending d.

Binary Codes and Empirical Evidence

Open Sets and Observation Sentences

- Note that observation sentences and elements of $\omega^{<\omega}$ are different!
- We will interpret observation sentences with some finite sequence of empirical data + any possible extrapolation into infinity.
- In other words, the open sets of the Baire space will be our observation sentences.

Laws of Nature as Duals of Observation Sentences

- Intuition: laws of nature may be refuted by *finite* means.
- If $\delta \in \omega^{\omega}$ and $d \in \omega^{<\omega}$ then δ is *consistent <u>with</u>* d if $d \triangleleft \delta$.

Bridging Language and Topology

A Propositional Language

$\mathcal{L} ::= \ p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \Diamond\phi$

Models and Semantics

Let $\mathfrak{M} = \langle \omega^\omega, \tau, [\![\ ]\!] \rangle$ be a topological model where (ω^ω, τ) is the Baire space and $[\![\ ]\!] \ : \ \mathcal{L} \rightarrow \wp(\omega^\omega)$ s.t.

- $[\![p]\!] = \{\delta \in \omega^\omega \ : \ p \text{ is true on } \delta\}$
- $[\![\neg\psi]\!] = W \setminus [\![\psi]\!]$
- $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$
- $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
- $[\![\phi \rightarrow \psi]\!] = [\![\neg\phi \vee \psi]\!]$
- $[\![\Diamond\varphi]\!] = \mathsf{cl}([\![\varphi]\!])$, $[\![\Box\varphi]\!] = \mathsf{int}([\![\varphi]\!])$

Bridging Language and Topology

Extension of Interpretation to Sets

Let $\Phi \subseteq \mathcal{L}$. Define $[\![\Phi]\!] := \bigcap_{\phi \in \Phi} [\![\phi]\!]$.

Truth Conditions

A formula φ is true on δ in $\mathfrak{M}$, i.e. $\mathfrak{M}, \delta \models \varphi$ iff $\delta \in [\![\varphi]\!]$.

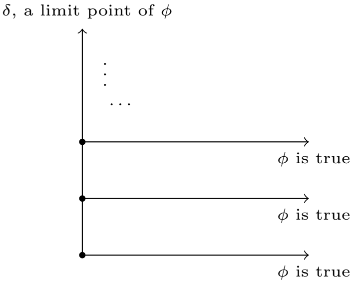
Finite Data Entailment

Take $d \in \omega^{<\omega}$. Then d *entails* ϕ or $d \gg \phi$ for short, if for all $\delta \in \omega^\omega$ s.t. $d \lhd \delta$ we find $\delta \in [\![\phi]\!]$.

Returning to (Laws of) Nature

Limits and Laws of Nature (cf. Schulte & Juhl, 1996)

Take $\delta \in \omega^\omega$ and $\phi \in \mathcal{L}$. We say that δ *is a limit point of* ϕ if ϕ is never refuted along δ, or alternatively, for every $d \in \omega^{<\omega}$ where $d \triangleleft \delta$ there is $\gamma \in \omega^\omega$ s.t. $d \triangleleft \gamma$ and ϕ is true on γ.



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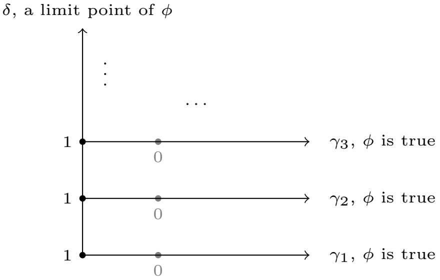

Example

Set 1 = “this swan is white” and 0 = ¬1.
So $\delta = 1^\omega$ (≈ “all swans are white”) is a limit point of
$[\![\phi]\!] = \{\gamma \in \omega^\omega \ : \ \exists n \in \omega(\gamma(n) = 0)\}$
(≈ “not all swans are white”).

Regaining Intuition

Laws of Nature (Closed Sets) in Terms of Limit Points

Call $\phi \in \mathcal{L}$ a *law of nature* if $[\![\phi]\!]$ contains all limit points of ϕ.

Example

If $\phi \approx$ "All swans are white" then $[\![\phi]\!]$ is closed but $\neg\phi$ is not because $\neg\phi$ is false on its limit point 1^ω.

Remark

Note that 1^ω is also a limit point of ϕ because $[\![\phi]\!] = \{1^\omega\}$. i.e. ϕ is true on 1^ω and hence never refuted along 1^ω.[a]

[a] For any $d \lhd \delta$ we always find $1^\omega \in \omega^\omega$ itself to make ϕ true.

Formalizing Falsification

Falsifiability and Never-ending Testability

Critical Tests for ϑ

Say that there is a *critical test* for ϑ if there is is $d \in \omega^{<\omega}$ s.t. for any δ where $d \triangleleft \delta$ we have $\delta \notin [\![\vartheta]\!]$.

(Scientific) Falsifiability of ϑ

Call ϑ (scientifically) *falsifiable* iff

- there is a critical test for ϑ;
- for all $d \in \omega^{<\omega}$ there is some $e \in \omega^{<\omega}$ s.t. $d \triangleleft e$ and for every $\varepsilon \in \omega^{\omega}$ where $e \triangleleft \varepsilon$ we have $\varepsilon \notin [\![\vartheta]\!]$.

Intuition

The more critical tests ϑ passes the more robust it seems.

Falsifiability and Never-ending Testability

Critical Tests for ϑ

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(Scientific) Falsifiability of ϑ

Call ϑ (scientifically) *falsifiable* iff

- there is a critical test for ϑ;
- $\forall d \in \omega^{<\omega}\ \exists e \in \omega^{<\omega}(d \lhd e)$ and $\forall \varepsilon \in \omega^{\omega}(e \lhd \varepsilon \to \varepsilon \notin [\![\vartheta]\!])$.

Distinction

Note that merely laws of nature $\neq$ falsifiable propositions.[a]

[a] Consider "no swan is gray, but at least one swan is black".

Generalizing Critical Tests (not Schulte et al.!)

From Critical Tests to Empirical Tests

Theories and Tests

- An *empirical theory* is $\langle \mathbb{O}, \mathbb{L} \rangle$ s.t. $\forall \phi \in \mathbb{O}$ and $\forall \psi \in \mathbb{L}$
  - $[\![\phi]\!]$ is an observation;
  - $\emptyset \neq [\![\psi]\!] \neq \omega^\omega$ is a law of nature.

  Set $\mathbb{T} := \mathbb{O} \cup \mathbb{L}$
- An *empirical test* is a triple $\langle \mathbb{T}, \pi, r \rangle$ consisting of
  - a theory $\mathbb{T}$.
  - a *falsifiable prediction* $\pi \in \mathcal{L}$ s.t. $[\![\mathbb{T}]\!] \subseteq [\![\pi]\!]$.
  - an *experimental result* $r \in \omega^{<\omega}$.

From Critical Tests to Empirical Tests

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  - an *experimental result* $r \in \omega^{<\omega}$.

Simplicity Assumptions

- Note that $\mathbb{T}$ is finite and **not** deductively closed.
- Only one (relevant) result.

Types of Falsification

Falsification (Formalized)

Call an empirical test $\langle \mathbb{T}, \pi, r \rangle$ *falsifying* if r is a critical test (result) for π s.t. for all $r \lhd \delta \in \omega^\omega$ we have $\delta \notin [\![\pi]\!]$.

Strong Falsification (Formalized)

Call an empirical test $\langle \mathbb{T}, \pi, r \rangle$ *strongly falsifying* if it is falsifying and there is precisely one $\vartheta \in \mathbb{L}$ s.t. $[\![\vartheta]\!] \subseteq [\![\pi]\!]$.[a]

[a] Note that then r is also a critical test (result) for ϑ.

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Weak Falsification (Formalized)

Call an empirical test $\langle \mathbb{T}, \pi, r \rangle$ *weakly falsifying* if it is falsifying and for $1 \leq i \leq n$ there are $\vartheta_i \in \mathbb{T}$ s.t. $\bigcap [\![\vartheta_i]\!] \subseteq [\![\pi]\!]$.

Positions in Philosophy of Science (Revisited)

Strong Falsification*ism*

For any $\mathbb{T}$ and π s.t. $[\![\mathbb{T}]\!] \subseteq [\![\vartheta]\!]$ we 'can' design an empirical test $\langle \mathbb{T}, \pi, r \rangle$ that is strongly falsifying.

Positions in Philosophy of Science (Revisited)

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Weak Falsification*ism*

For any $\mathbb{T}$ and π s.t $[\![\mathbb{T}]\!] \subseteq [\![\pi]\!]$, the empirical test $\langle \mathbb{T}, \pi, r \rangle$ is merely/at best weakly falsifying.

Outlook

Open Questions

- Relation of falsification(ism)s to separation axioms?
- Different falsification(ism)s $\stackrel{?}{\to}$ Different operators?
- ...

Other research

- A lot of work within the ILLC, closer to modal logic (than our approach)
- Evidence and multi-agent epistemology
- van Benthem & Sarenac (2004)
- Baltag et al. (2022)

Sources

Özgün, A. (2013). Topological models for belief and belief revision. Master's thesis, ILLC, University of Amsterdam.
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